J. Jpn. Soc. Comp. Statist., **xx**(200x), 1–3

COMPUTATION OF THE TEST STATISTIC AND THE NULL DISTRIBUTION IN THE MULTIVARIATE ANALOGUE OF THE ONE-SIDED TEST

Yoshiro Yamamoto*, Akio Kudô† and Katsumi Ujiie‡

ABSTRACT

The multivariate analogue of the one sided test derived in Kudô (1963) is considered. A handy method of computing the test statistic and its significance probability is given. The method is based on applying sweep out operations on a certain matrix in a systematic manner and applying the Fortran subroutine of Sun (1988).

1. Introduction

The problem of this paper is as follows. Given a p-variate normal distribution: $N_p(\boldsymbol{\theta}, \boldsymbol{\Lambda})$, we consider the testing problem: $\mathrm{H}_0 : \boldsymbol{\theta} = \mathbf{0}$ versus $\mathrm{H}_1 - \mathrm{H}_0 : \boldsymbol{\theta} \geq \mathbf{0}$, where $\boldsymbol{\Lambda}$ is assumed to be known and $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_p)' \geq \mathbf{0}$ means Max $\theta_i > 0$, Min $\theta_i \geq 0$, thus the space of the alternative hypothesis is a cone. The MLE of $\boldsymbol{\theta}$ in the space of $\mathrm{H}_0 \cup \mathrm{H}_1$ is often called "the projection into the cone" because of the very nature of the alternative hypothesis. In this paper we also set the hypothesis $\mathrm{H}_2 : \boldsymbol{\theta} \neq \mathbf{0}$, and consider a testing problem : H_1 versus $\mathrm{H}_2 - \mathrm{H}_1$.

More than 3 decades have passed since the paper "Multivariate Analogue of the One-sided Test" was published in Biometrika where the above problem was treated and the likelihood ratio test was derived. This paper has been criticized in two aspects. Difficulties in computing (i) the test statistic and (ii) the significance probability. (See for instance Tang *et al.* (1989). See also Barlow *et al.* (1972) and Robertson *et al.* (1988).)

Our purpose is to demonstrate methods to compute these two. In Section 2 we quickly review the outline of the problem, and Section 3 is devoted to the method for computing the MLE, which will find its application in computing an optimum linear test given in Shi *et al.* (1987), and Section 4 to a numerical example. In Section 5 an application of Sun's subroutine is demonstrated and its accuracy examined, and the final Section is for concluding discussions.

The first of two distribution functions given in Section 2 was given by Kudô (1963) and the second can be derived from Theorem 2.3.1. of Robertson *et al.* (1988). The second distribution was treated in Robertson and Wegman (1978) and we documented the same in the present paper's form.

*Graduate School of Natural Science and Technology, Okayama University, 3-1-1, Tsushima-naka, Okayama 700-0082, Japan E-mail: `yamamoto@xxx.okayama-u.ac.jp`

†Faculty of Economics and Information Science, Hyogo University, 2301, Shinzaike, Hiraoka-Cho, Kakogawa 675-0101, Japan

‡Department of Mathematics, School of Science, Tokai University, 1117, Kitakaname, Hiratsuka 259-1292, Japan

Key words: Level probability; Order restricted statistical inference; Orthant probability

1.1. How to use the `.cls` or `.sty` files

At first, place appropreate class/style file in the same directory with your TEX document(s).

- `jscs_sub.cls`: LATEX 2ε, for submission
- `jscs.cls`: LATEX 2ε, for the final image
- `jscs_sub.sty`: LATEX2.09, for submission
- `jscs.sty`: LATEX2.09, for the final image

If you use LATEX 2ε, you should begin your TEX document with

```
\documentclass{jscs_sub}
```

(for submission), or

```
\documentclass{jscs}
```

(for the final image). Or if you use LATEX2.09, you should begin with

```
\documentstyle[jscs_sub]{article}
```

(for submission), or

```
\documentstyle[jscs]{article}
```

(for the final image).

We already defined some environment for theorems, propositions, *etc.* See this TEX source file and the following examples.

Lemma 1 *This is an example of the lemma environment.*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}. \tag{1}$$

Theorem 1 *This is an example of the theorem environment.*

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}. \tag{2}$$

In theorem environment, the reference to an equation(s) should not be italicized. Use `\rm` *command (LATEX2.09) or* `\textup{}` *command (LATEX 2ε) like "From the equation* (1)*, …", "According to the conditions* (i)–(iv)*, …",* etc.

Proof. In our `.cls` and `.sty` files, `\proof` command and `\qed` command are defined. You can use `\proof` for opening the proof and `\qed` for closing the proof. □

Table 1: The first $2^p - 1$ binary numbers

No.	Binary No.	Sweep out position
1	1	$1000\cdots0$
2	10	$0100\cdots0$
3	11	$1100\cdots0$
4	100	$0010\cdots0$
$\vdots$	$\vdots$	$\vdots$
2^{p-2}	$1\cdots1$	$111\cdots10$
$2^{p-2}+1$	$10\cdots0$	$000\cdots01$
$\vdots$	$\vdots$	$\vdots$
2^{p-1}	$11\cdots1$	$111\cdots11$

2. A brief review

As the variance matrix Λ is assumed to be known, we can assume there is only one observation x. Let P be the set of integers $P = \{1, \cdots, p\}$ and $M = \{i_1, \cdots, i_m\}$ be a subset of P. It was shown that there exists uniquely a set satisfying the condition stated below. (see Beale (1959))

Without loss of generality, we state the condition in the case when the set is the first m natural numbers: $M = \{1, \cdots, m\}$ where $m \leq p$.

Let the observation vector $\boldsymbol{x}' = (x_1, \cdots, x_p)$ be partitioned into m and $p-m$ components and the mean vector and the variance matrix be partitioned accordingly:

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_{(1)} \\ \boldsymbol{x}_{(2)} \end{pmatrix}, \quad \boldsymbol{\theta} = \begin{pmatrix} \boldsymbol{\theta}_{(1)} \\ \boldsymbol{\theta}_{(2)} \end{pmatrix}, \quad \boldsymbol{\Lambda} = \begin{pmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} \\ \boldsymbol{\Lambda}_{21} & \boldsymbol{\Lambda}_{22} \end{pmatrix}.$$

There exists uniquely the subset M satisfying the condition.

$$\left\{ \begin{array}{rcl} \boldsymbol{\Lambda}_{11}^{-1}\boldsymbol{x}_{(1)} & < & 0 \\ \boldsymbol{x}_{(2)} - \boldsymbol{\Lambda}_{21}\boldsymbol{\Lambda}_{11}^{-1}\boldsymbol{x}_{(1)} & \geq & 0 \end{array} \right. \tag{3}$$

When $m = 0$, $M = \phi$, the empty set, and when $m = p$, the second condition in (3) is vacuous.

Table 1 is inserted here.

REFERENCES

Akaike, M. (1973). Information theory and extension of the maximum likelihood principle. In *Second International Symposium of Information Theory*, N. B. Petrov and F. Czaki (eds.), 267–281, Budapest: Akademiai Kiado.

Beale, E. M. L. (1959). On quadratic programming. *Naval Research Logistic Quarterly* **6**, 227–243.

Miyahara, H. and Tango, T. (eds.) (1995). *Handbook of Medical Statistics.* Tokyo: Asakura Shoten. (in Japanese)

Swayne, D. F., Cook, D. and Buja, A. (1998). Xgobi: Interactive dynamic data visualization in the X Window System. *Journal of Computational and Graphical Statistics* **7**, 113–130. See also `http://www.research.att.com/areas/stat/xgobi/`.

Tarumi, T. and Kudô, A. (1974). An algorithm related to all possible regression and discriminate analysis. *Journal of the Japan Statistical Society* **4**, 47–50.